Graph Theory and Combinatorics

M. Math. II

Semestral Examination

Instructions: All questions carry ten marks. Results proved in the course can be used without proof.

1. Let $n \geq 3$ be a natural number and let S be a subset of n points in the plane such that the distance between any two distinct points of S is at least one. Then, prove that there are at most $3n - 6$ pairs u, v in S such that $d(u, v) = 1$.

2. Let G be a simple bipartite graph with minimum degree $\delta(G)$ and maximum degree $\Delta(G)$. Prove that the edges of G can be coloured with $\delta(G)$ colours in such a way that all colours appear on edges incident with any vertex and that edges of G can be coloured with $\delta(G)$ colours in such a way that no colour is repeated amongst edges that are incident with a vertex.

3. Prove that in a Steiner system $S(2, k, v)$ with $b > v$, we must have $v \geq k^2$. Further prove that, in case $v = k^2$, the set of blocks can be partitioned into $k+1$ "parallel classes" of k blocks such that the blocks in a given parallel class are pairwise disjoint.

4. Find binary self-dual codes of length 4, 6, and 8.